Street-Routing Problems

Lecture 26
Sections 5.1 - 5.2

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Outline

Street-Routing Problems

- There are many problems that fall under the heading "Street-Routing Problems."
- We will consider five of them.

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  - The Mail Carrier Problem
  - The Königsberg Bridge Problem (famous)
  - The Bridges of Madison County

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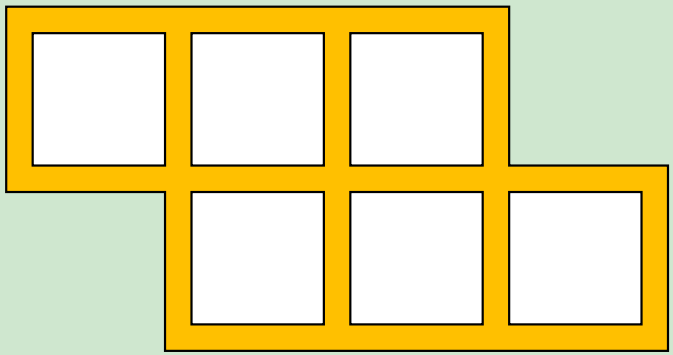
- We will consider five of them.
  - The Security Guard Problem
  - The Mail Carrier Problem
  - The Königsberg Bridge Problem (famous)
  - The Bridges of Madison County
  - The Traveling Salesman Problem (famous)

Example

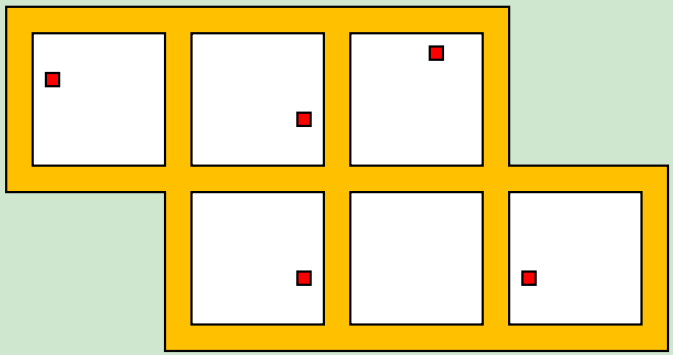
Example (The Security Guard and Mail Carrier Problems)



- In the Security Guard Problem, we want route that travels every hallway (street) *once* and has the minimal total length.

Example

Example (The Security Guard and Mail Carrier Problems)



- In the Mail Carrier Problem, we want a route that every every street *twice* (except the boundary streets) and has the minimal total length.

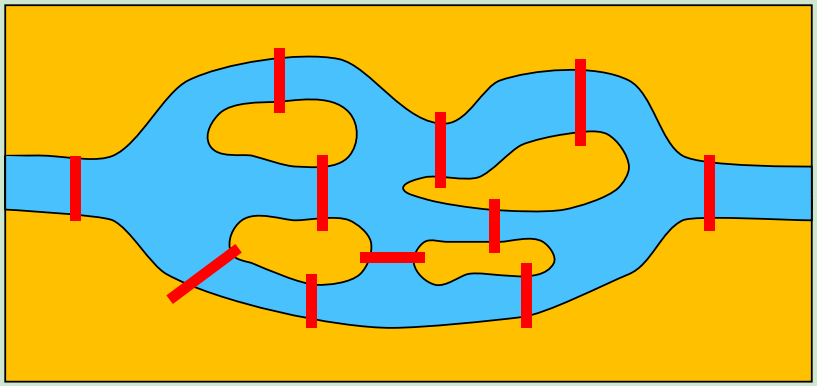
The Königsberg Problem

Example (The Königsberg Problem)

- A Königsberger would like to take a stroll across the seven bridges of Königsberg.
- Can it be done without ever crossing the same bridge twice?
- Does it matter where the stroller starts?

The Bridges of Madison County Problem

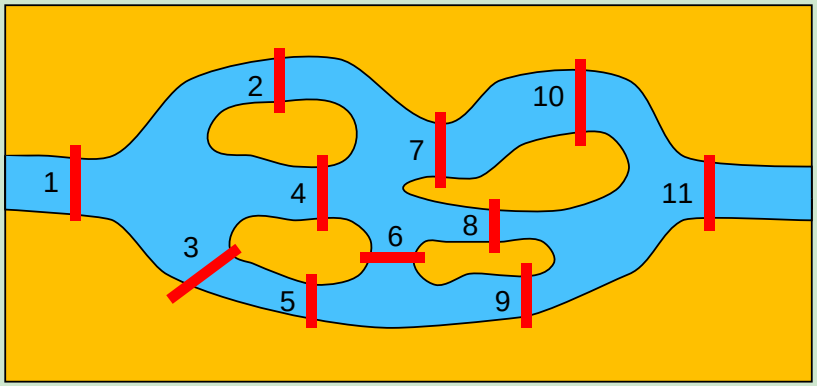
Example (The Bridges of Madison County Problem)



- A photographer wants to photograph each of the 11 bridges of Madison County.
- He must cross the bridge to photograph it and each bridge has a $5 toll.
- What route will minimize the total cost?

The Bridges of Madison County Problem

Example (The Bridges of Madison County Problem)



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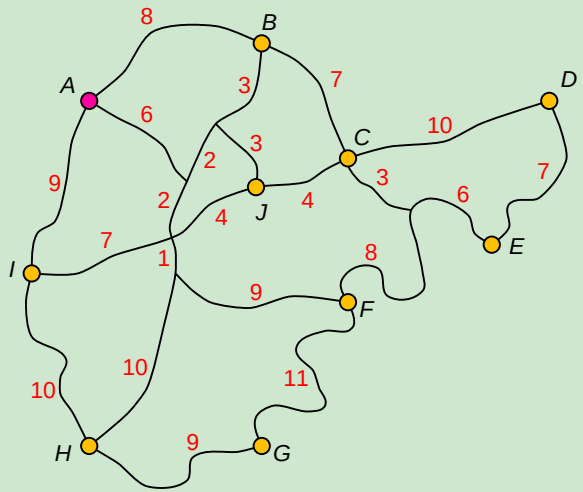
The Traveling Salesman Problem

Example (The Traveling Salesman Problem)



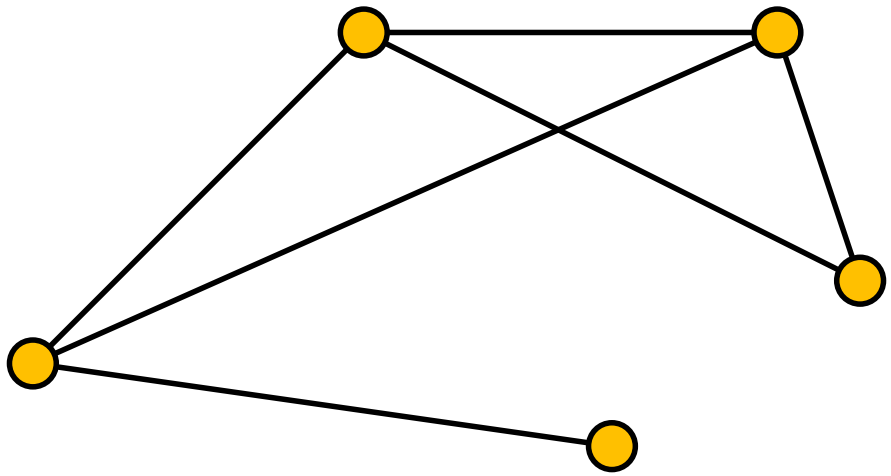
- A salesman is located in a city.
- He must make a trip during which he visits each of a number of other cities and return to his home city.
- He knows the distance from every city to every other city.
- What route will minimize the total distance traveled?

Outline

Definitions

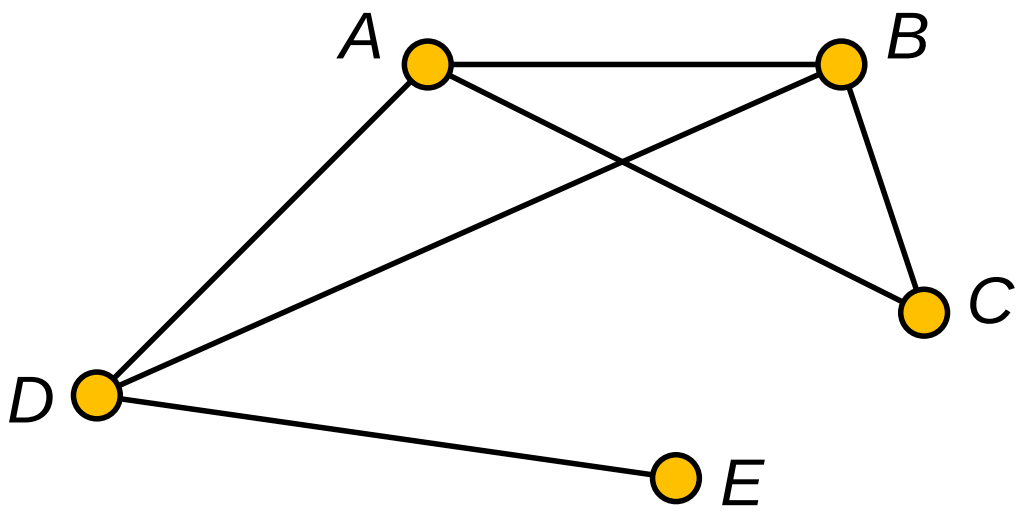
Definition

A graph is a collection of vertices and edges. We normally draw the vertices as dots and the edges as lines. Each edge connects a pair of vertices.



Definitions

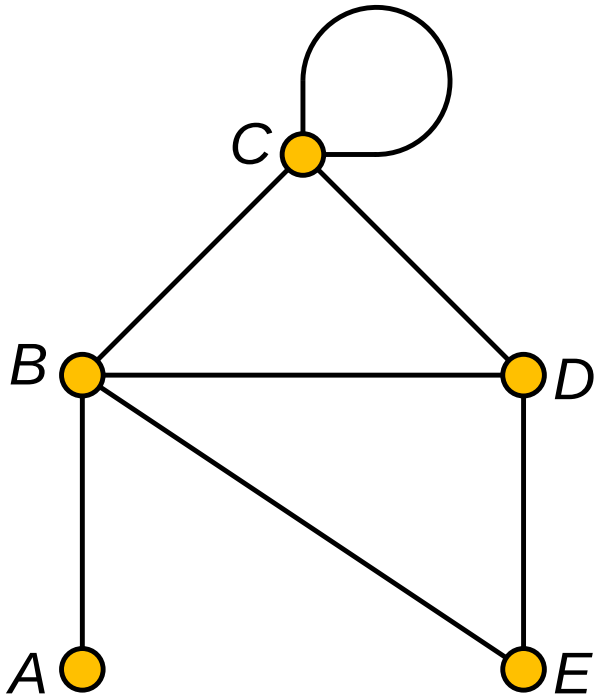
- We can give the vertices labels, e.g., A, B, C, etc.
- Then use those labels to identify the edges, e.g., AB, AC, etc.



Adjacent Vertices

Definition

Two vertices are adjacent if they are connected by an edge.



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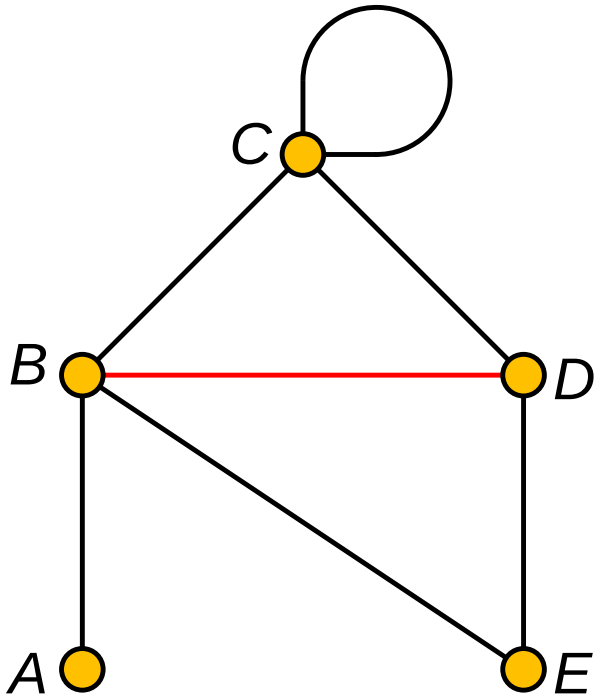

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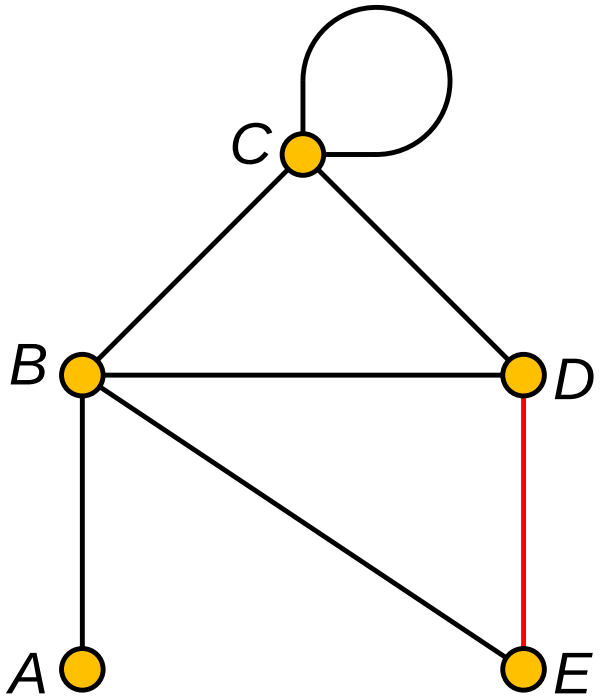

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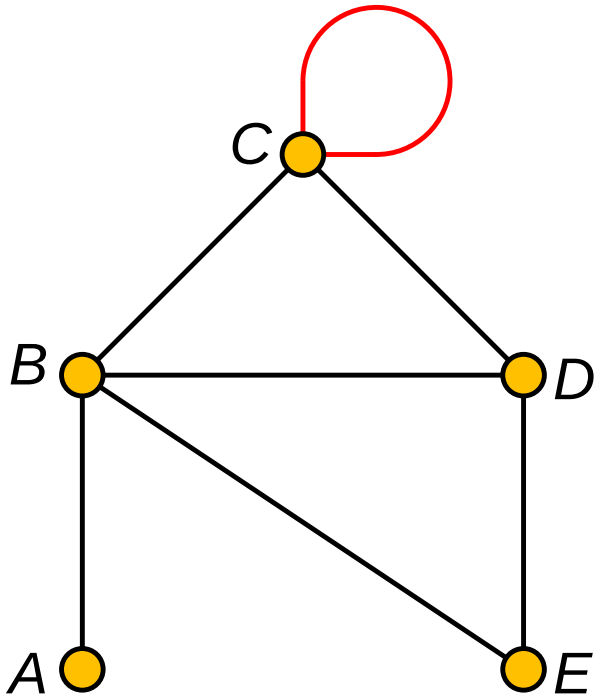
Two vertices are adjacent if they are connected by an edge. If a vertex has an edge connected to itself, that edge is also called a loop.



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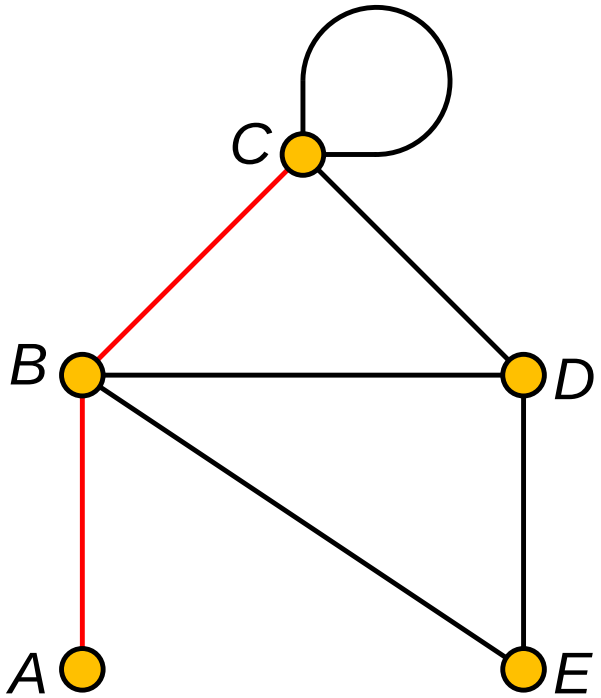

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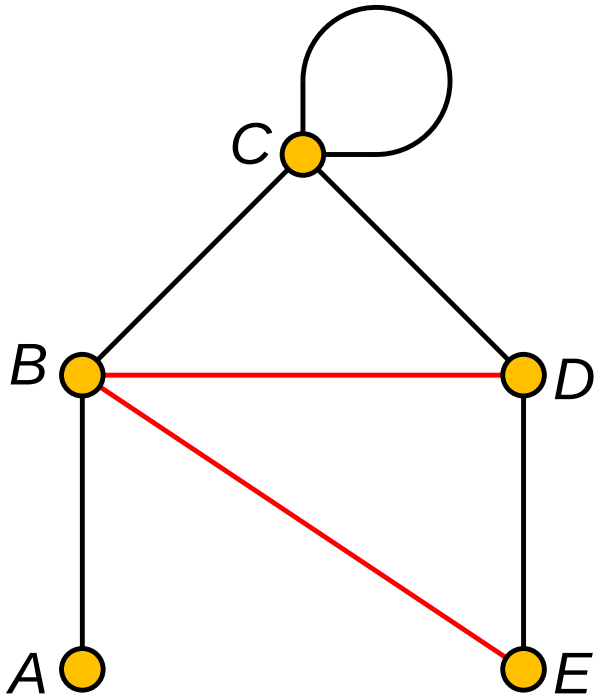
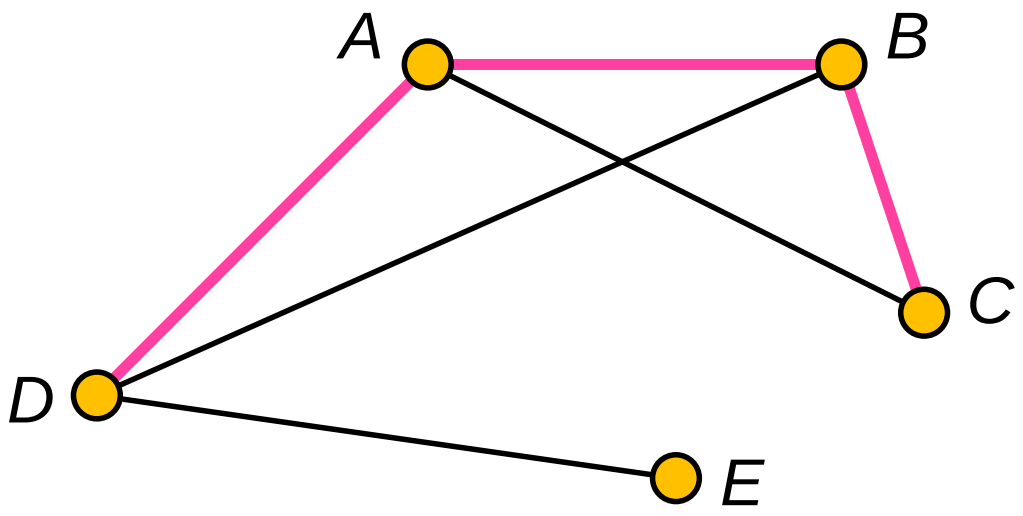


Definitions

Definition (Path)

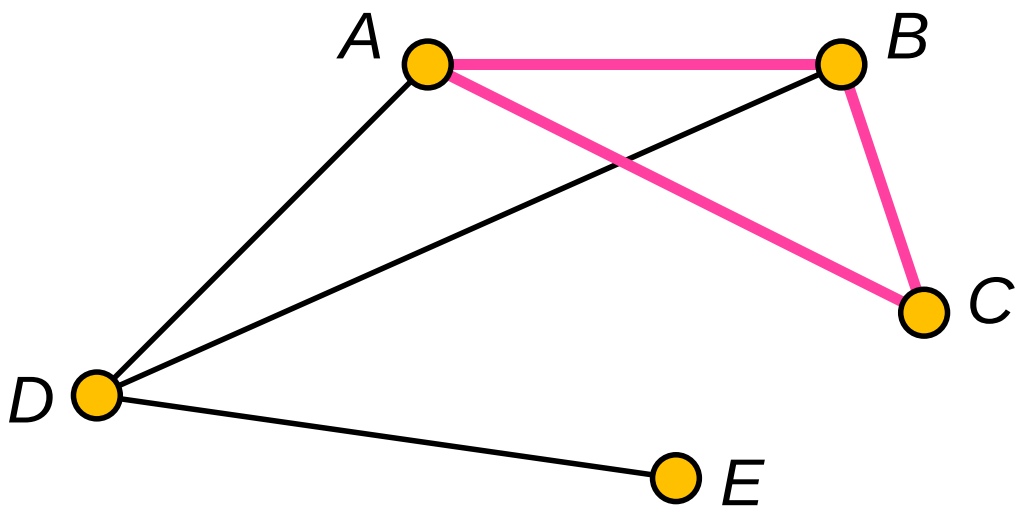
A path is a sequence of distinct adjacent edges, each edge adjacent to the next edge. We may denote a path by listing the vertices through which it passes, e.g., *DABC*.



Definitions

Definition (Circuit)

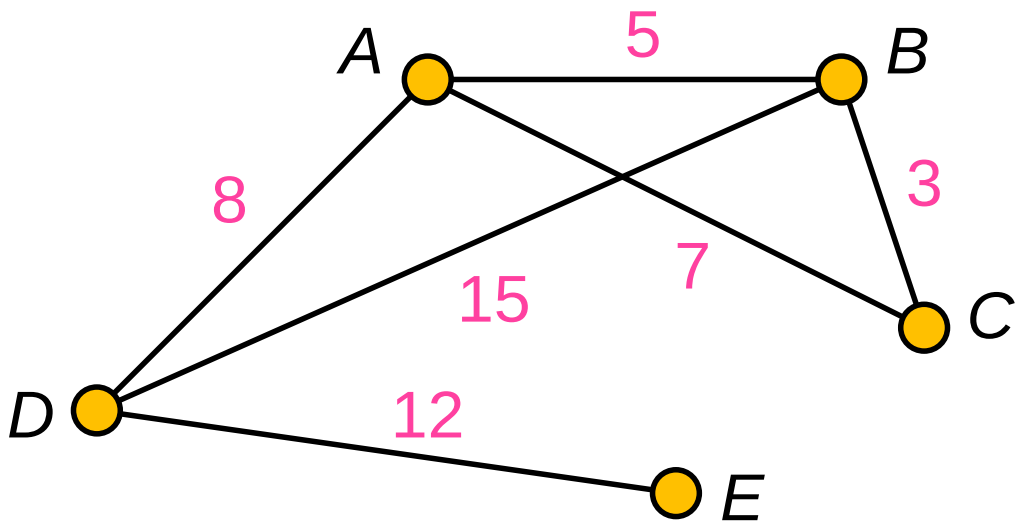
A circuit is a path that begins and ends at the same vertex, e.g., *ABCA*.



Definitions

Definition (Weighted Graph)

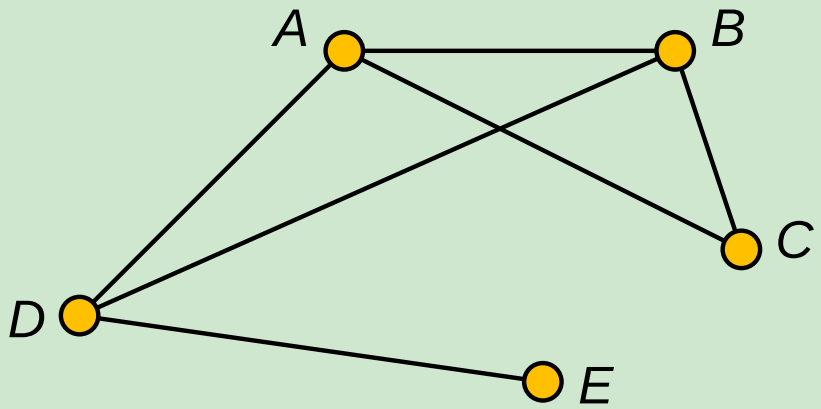
A weighted graph is a graph in which every edge is assigned a value (its weight).



Outline

Social Networks

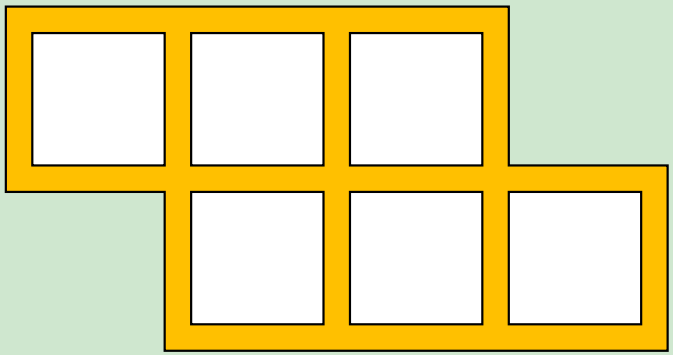
Example (Social Networks)



- Andy, Bob, Chuck, Dave, and Eddie all belong to a social network.
  - Andy is friends with Bob, Chuck, and Dave (and vice versa).
  - Bob is friends with Chuck and Dave (and vice versa).
  - Dave is friends with Eddie (and vice versa).

Example

Example (The Security Guard and Mail Carrier Problems)



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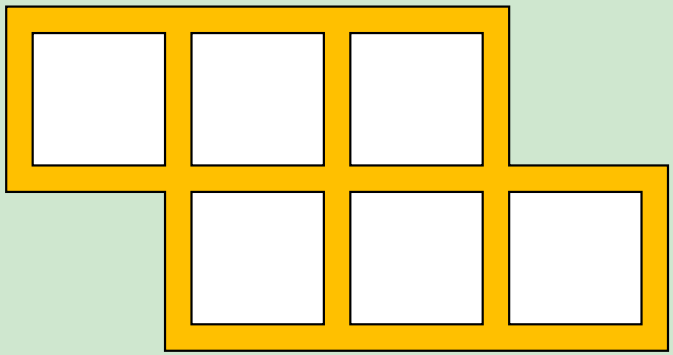

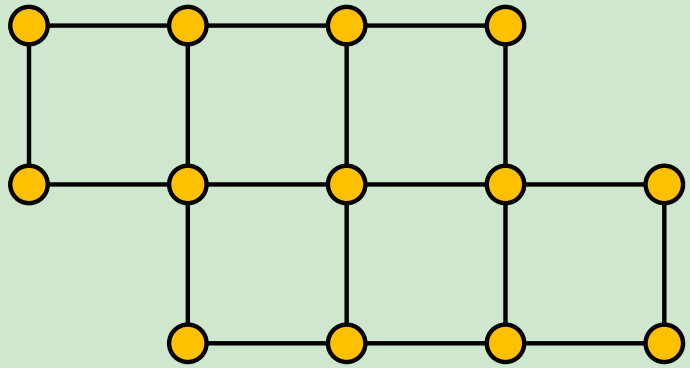

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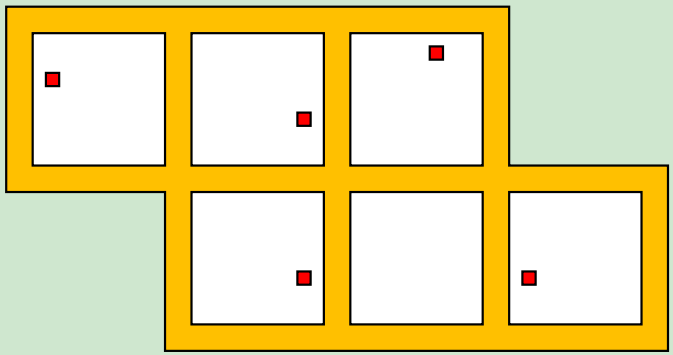

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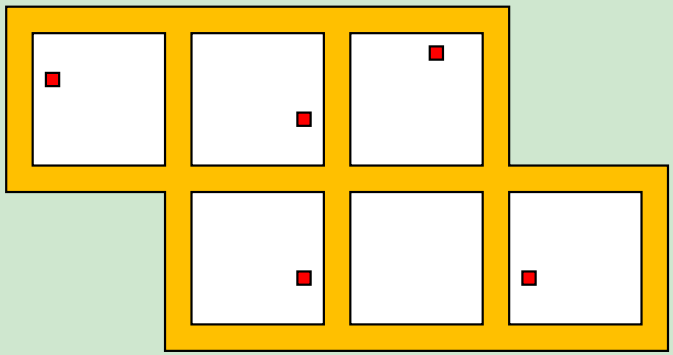
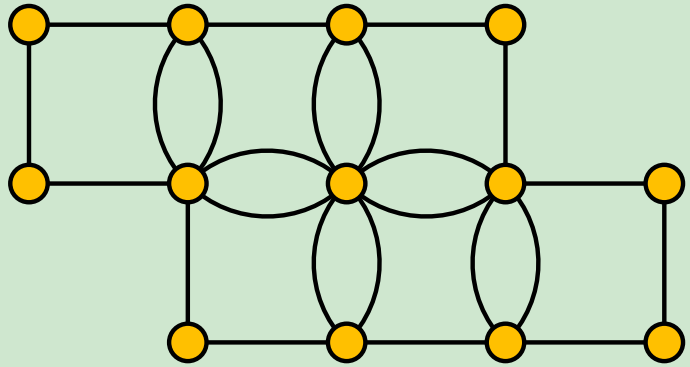
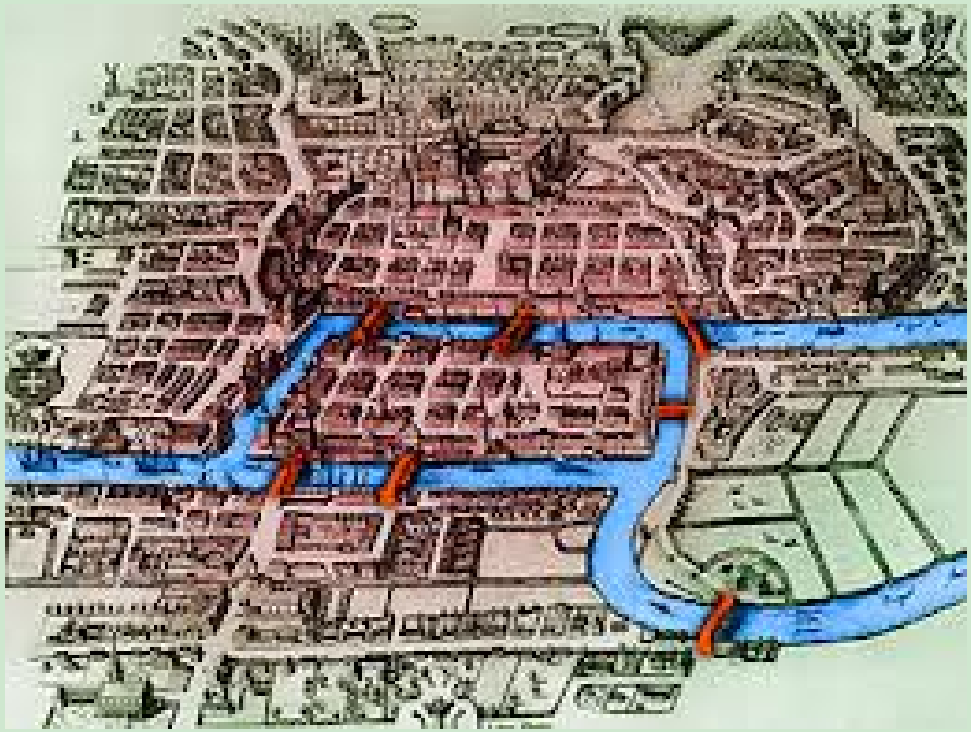




- In the Mail Carrier Problem, we want a “path” that traverses every edge *at least twice* (except the boundary edges once) and has the minimal total length.
- With the duplicated edges, the Mail Carrier Problem is the same as the Security Guard Problem.

Example

Example (The Bridges of Königsberg Problem)



- In the Bridges of Königsberg Problem, we want a circuit that traverses each edge *exactly once*.

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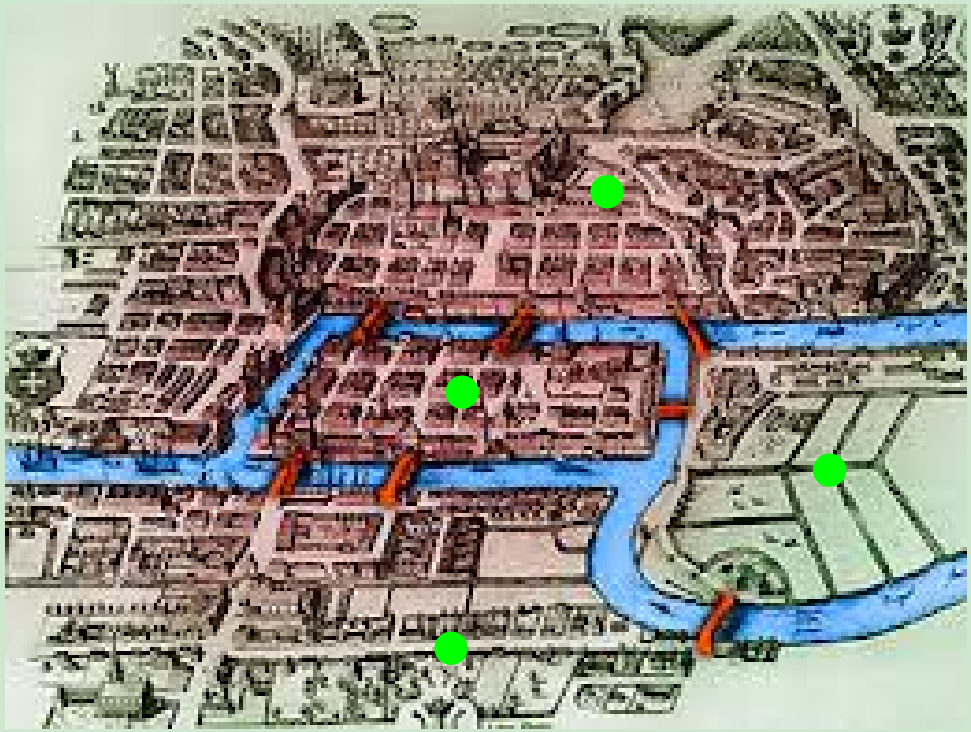

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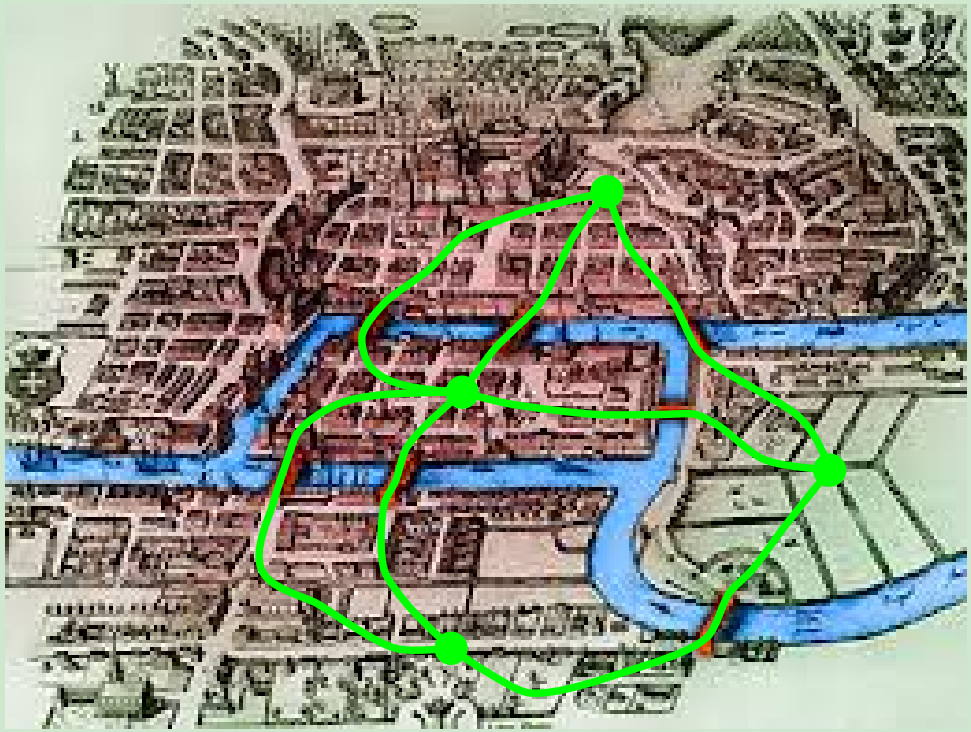

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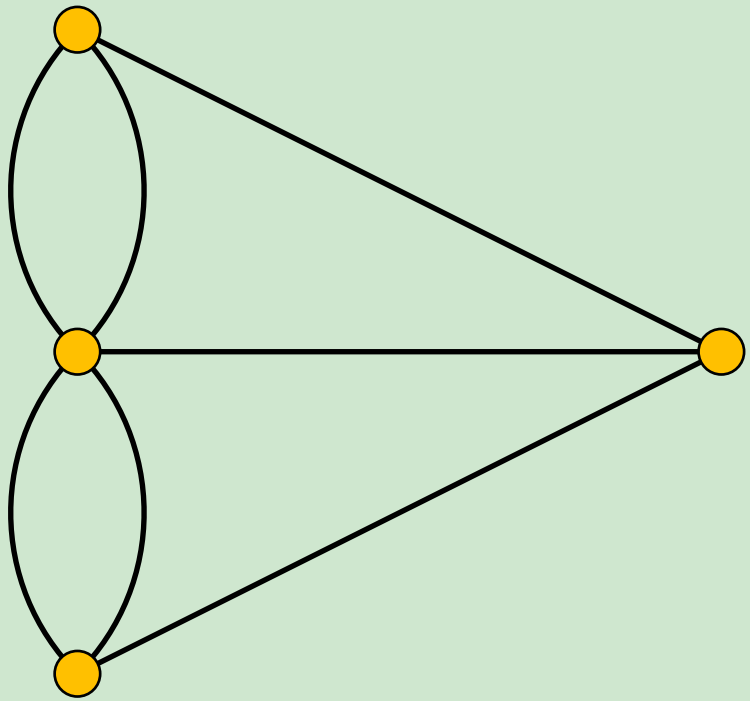
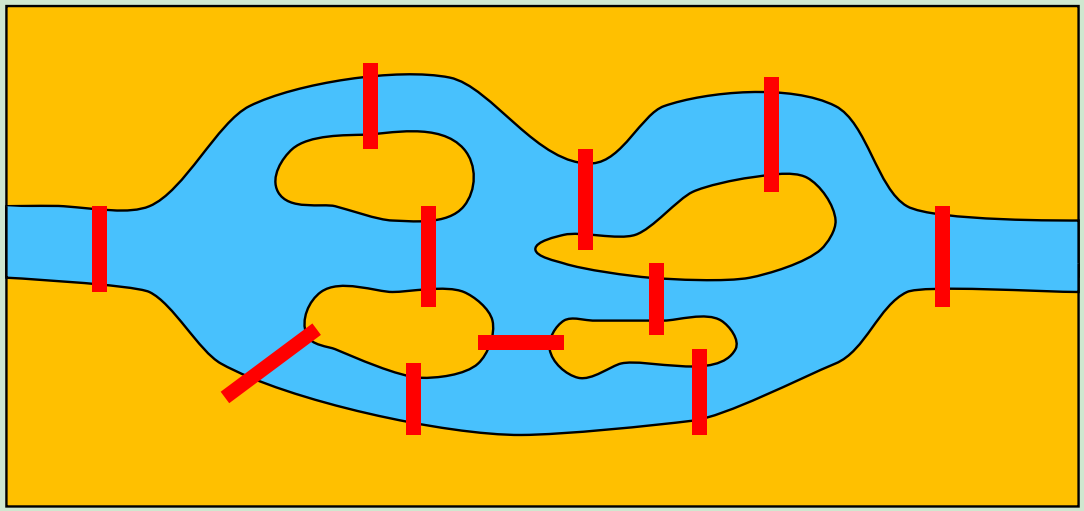


- We draw a graph that shows only the relevant parts.

Example

Example (The Bridges of Madison County Problem)



- In the Bridges of Madison County Problem, we want a circuit that traverses each edge *at least once* and has the minimal total length.
- This is the same as the Security Guard Problem.

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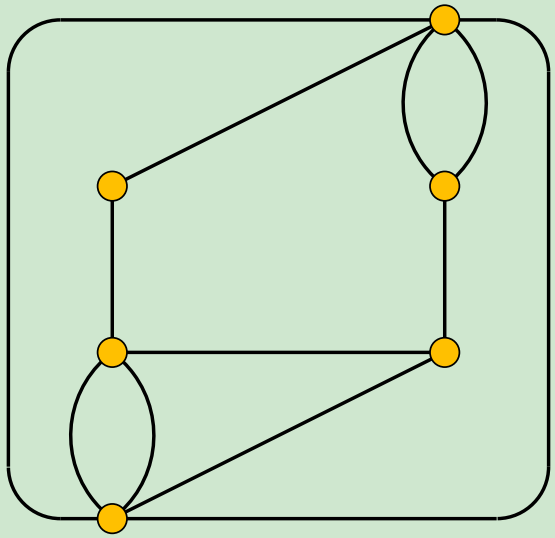

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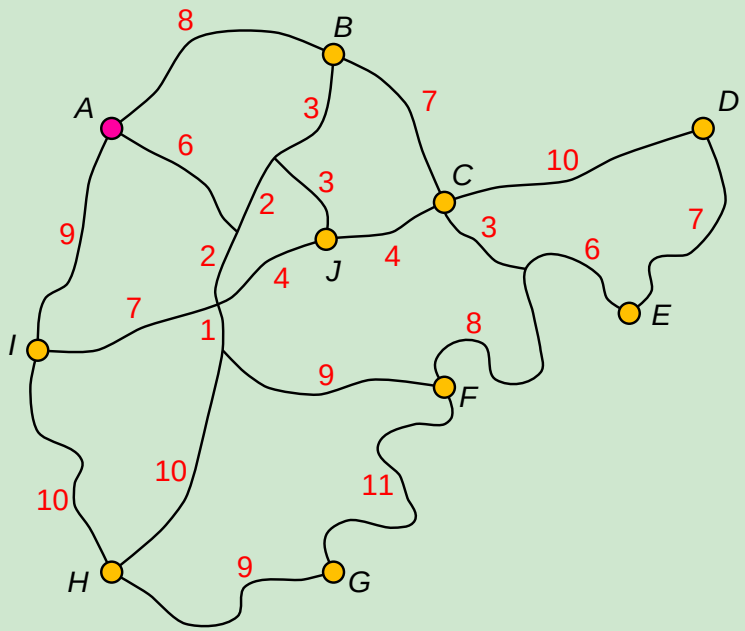

- In the Traveling Salesman Problem, we want a circuit that visits each vertex *at least once* and has the minimal total length.

Example

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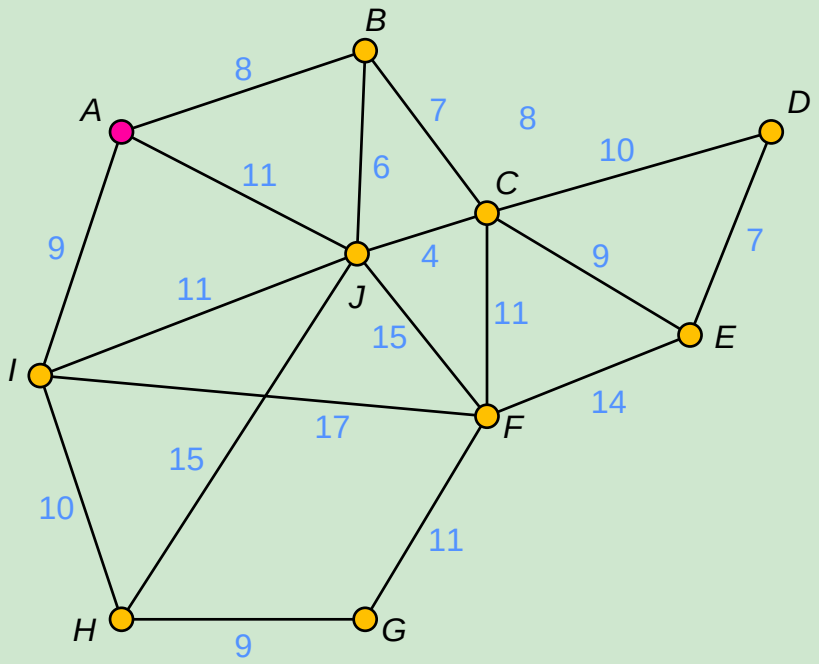

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Traveling Salesman Map

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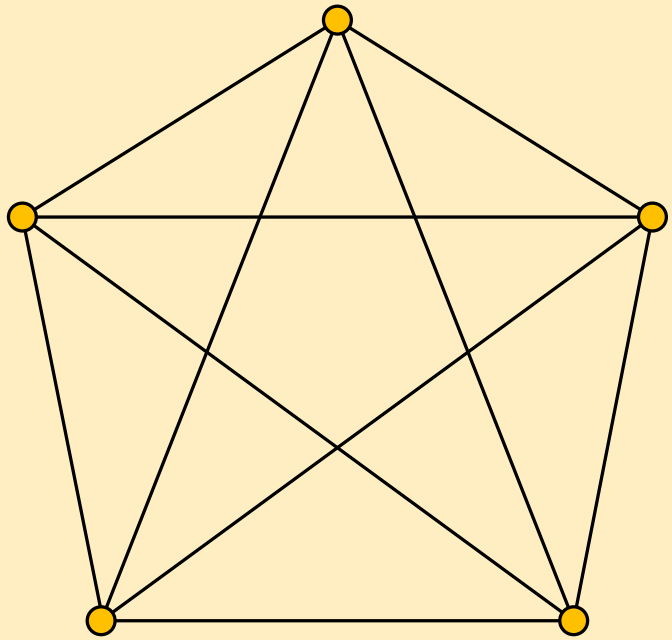

- With few vertices, we may draw the complete graph.

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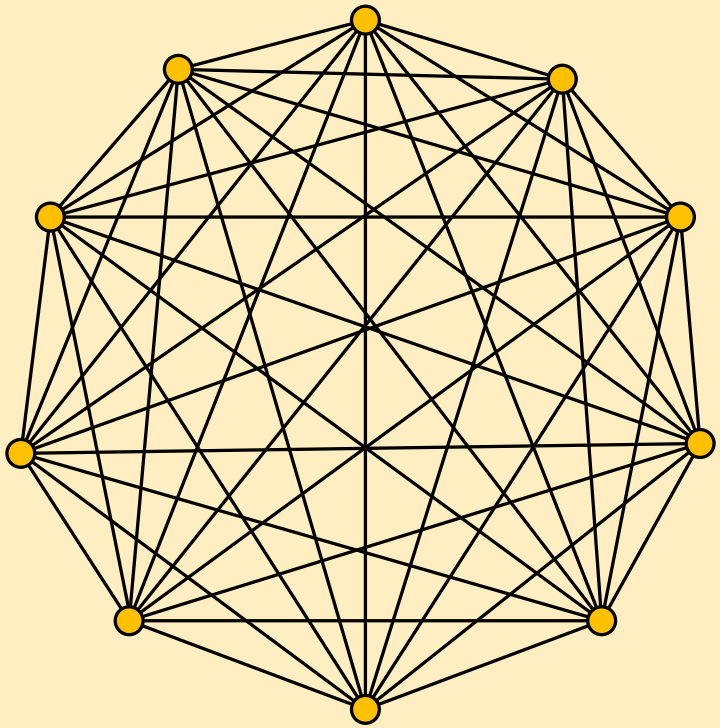

- With few vertices, we may draw the complete graph.
- But with many vertices, that is not practical.

Outline

Assignment

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- Chapter 5: Exercises 2, 3, 7, 13, 15, 19, 20, 21, 22, 27.